Article

Effectiveness of the Concrete- Representational- Abstract (CRA) Strategy In Improving the Basic Numeracy Skills of Learners with Learning Disability

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Abstract

This study examined the effectiveness of the Concrete–Representational–Abstract (CRA) strategy in improving the basic numeracy skills of learners with learning disabilities. Specifically, it aimed to determine learners' numeracy performance before and after the implementation of the CRA strategy, focusing on number sense, basic operations, and problem-solving skills. A quasi-experimental pretest–posttest research design was employed to measure the impact of the intervention. The participants consisted of identified learners with learning disabilities who demonstrated difficulties in basic numeracy. Purposive sampling was used to ensure that the intervention addressed learners with documented numeracy challenges. Data were gathered using a researcher-developed numeracy assessment administered before and after the CRA-based instruction. The intervention involved systematic instruction progressing from the use of concrete manipulatives, to visual and pictorial representations, and finally to abstract mathematical symbols. The results revealed a significant improvement in learners' overall numeracy performance after the implementation of the CRA strategy. Notable gains were observed in number sense and basic operations, indicating that learners developed a clearer conceptual understanding of mathematical relationships and procedures. Improvements were also evident in problem-solving skills, suggesting increased ability to apply mathematical knowledge to practical tasks. These findings affirm that CRA effectively bridges conceptual understanding and procedural fluency for learners with learning disabilities. The study concludes that the CRA strategy is a research-based and inclusive instructional approach that supports meaningful numeracy development. It recommends the sustained and systematic use of CRA in numeracy instruction, supported by adequate instructional resources, teacher training, and continuous progress monitoring to maximize learning outcomes for learners with learning disabilities.

Keywords: Concrete–Representational–Abstract strategy, basic numeracy, learning disabilities, inclusive education, mathematics intervention

Introduction

Learners with learning disabilities (LD) often experience persistent difficulty in developing basic numeracy skills such as number sense, place value, and the accurate and fluent use of basic operations. These difficulties are not simply a matter of "low ability" but are frequently linked to challenges in working memory, processing speed, and conceptual understanding factors that can make traditional, symbol-heavy mathematics instruction less accessible. Because numeracy is foundational for higher mathematics and everyday problem-solving, schools need intervention approaches that explicitly build understanding while reducing cognitive load and supporting gradual movement from concrete meaning to abstract symbols. One approach that has gained strong attention in special education is the Concrete–Representational–Abstract (CRA) strategy, which teaches concepts through a structured progression: hands-on materials (concrete), visual/pictorial models (representational), and finally numbers and symbols (abstract).

The CRA strategy is widely recognized as a form of explicit, systematic instruction that supports learners who struggle with mathematics by making ideas visible and "touchable" before expecting mastery of abstract notation. CRA is particularly relevant for learners with LD because it provides repeated opportunities to connect conceptual meaning (what quantities represent) with procedural steps (how to compute). In many classrooms, however, learners with LD are still expected to perform abstract computations early, often without enough scaffolded experiences using manipulatives and visual models. This gap between instructional demands and learner needs contributes to low performance, math anxiety, and disengagement. As a result, determining the effectiveness of CRA in improving basic numeracy skills among learners with LD is important for strengthening intervention planning, inclusive instruction, and evidence-based special education practice.

Review of Related Literature and Studies

Research has repeatedly shown that CRA can improve mathematical performance for learners with LD and those with significant math difficulties. Early work by Witzel, Mercer, and Miller demonstrated that explicit instruction models that incorporate CRA-related sequencing can support learners with math difficulties in algebra contexts, showing gains when instruction emphasized structured progression and clarity. Witzel (2005) further reported positive outcomes when CRA was used to teach algebra to students with math difficulties in inclusive settings, providing evidence that structured representational supports can strengthen understanding beyond rote procedures. Extending CRA to secondary learners, Witzel, Riccomini, and Schneider documented practical implementation of CRA with secondary students with learning disabilities, highlighting CRA's role in scaffolding understanding through concrete and representational stages.

At the level of foundational skills, Flores (2010) found that the CRA instructional sequence improved subtraction performance (including regrouping) among students at risk for failure, supporting the use of CRA for basic computation skills that are closely tied to numeracy development. Butler and colleagues (2003) compared a CRA sequence with a representational–abstract sequence in teaching fractions to students with mathematics disabilities and found stronger overall outcomes for learners receiving CRA, suggesting that concrete experiences can provide added leverage for learners with disabilities.

More recent studies extend CRA into applied and functional mathematics. Bouck et al. (2017) explored CRA instruction for students with disabilities in solving coin "change-making" problems and reported positive effects, indicating CRA's usefulness beyond traditional computation into functional

numeracy. Contemporary research continues to show CRA's promise across different mathematical topics and learner profiles. For example, Al-salahat (2022) reported positive effects of CRA sequential lessons for students with mathematics learning difficulties, including improvements in mathematical problem-solving tasks. Kaya (2023) also found CRA to be effective in teaching basic addition and subtraction-related skills to a student with autism in an inclusive setting, further supporting CRA's adaptability for learners with diverse needs.

At the instructional systems level, intervention-oriented research and summaries emphasize that CRA is most effective when delivered explicitly and consistently, with strong links between concrete experiences, visual representations, and symbolic reasoning. More recent inclusion-oriented work also highlights that CRA can strengthen retention and conceptual development in secondary mathematics contexts when manipulatives and representations are intentionally used. Finally, guidance resources and compilations of CRA research continue to cite the repeated demonstrations of CRA effectiveness for students with disabilities across mathematics domains, supporting its relevance for numeracy interventions.

Although the literature provides substantial support for CRA in improving mathematics outcomes among learners with LD and related learning difficulties, several gaps remain. First, many well-cited CRA studies focus on specific topics (e.g., algebra, fractions, regrouping, or functional money skills) rather than targeting basic numeracy as an integrated set of foundational competencies (number sense, place value, and basic operations combined). Second, even when CRA is shown to be effective, fewer studies foreground the intervention specifically for learners formally identified with learning disabilities, particularly in school-based settings where teachers must balance inclusive classroom demands, limited resources, and varied learner profiles. Third, there remains a need for context-specific research that examines CRA's effectiveness as a structured strategy for strengthening basic numeracy outcomes among learners with LD especially in settings where numeracy gaps are persistent and where evidence can directly inform intervention programs and inclusive education practice. This study addresses these gaps by evaluating CRA's effectiveness in improving the basic numeracy skills of learners with learning disabilities.

Research Objectives

This study aims to determine the effectiveness of the Concrete–Representational–Abstract (CRA) strategy in improving the basic numeracy skills of learners with learning disabilities. Specifically, it seeks to assess the learners' level of basic numeracy skills before and after the implementation of the CRA strategy, focusing on number sense, basic operations, and problem-solving abilities. The study also aims to examine the extent to which the CRA strategy enhances learners' understanding of mathematical concepts through the use of concrete materials, visual representations, and abstract symbols. In addition, it seeks to explore teachers' observations and perceptions regarding the use of the CRA strategy in addressing the learning needs of learners with learning disabilities. Finally, the study aims to identify challenges encountered in implementing the CRA strategy and to generate evidence-based insights that may inform instructional planning, intervention programs, and inclusive education practices for learners with learning disabilities.

Research Methodology

This study employed a quasi-experimental research design to determine the effectiveness of the Concrete–Representational–Abstract (CRA) strategy in improving the basic numeracy skills of learners with learning disabilities. The quasi-experimental approach was appropriate because the

study aimed to examine changes in learners' numeracy performance before and after the implementation of the CRA strategy without random assignment of participants. This design allowed for a systematic comparison of learners' numeracy skills across instructional phases.

The participants of the study consisted of identified learners with learning disabilities enrolled in the selected grade level. Purposive sampling was used to select participants who had documented difficulties in basic numeracy skills and were receiving or eligible for special education support. This ensured that the intervention directly addressed the needs of learners with learning disabilities.

Data were collected using a researcher-developed numeracy assessment that measured key areas of basic numeracy, including number sense, basic operations, and simple problem-solving skills. A pretest–posttest procedure was employed to assess learners' performance before and after the implementation of the CRA strategy. The intervention was implemented over a specified instructional period, during which learners were taught using a structured CRA sequence that began with hands-on manipulatives (concrete), progressed to pictorial and visual representations (representational), and culminated in the use of mathematical symbols and equations (abstract).

Data analysis involved the use of descriptive and inferential statistics to determine changes in learners' numeracy performance. Ethical considerations were strictly observed, including obtaining informed consent from parents or guardians, ensuring confidentiality of learner data, and using assessment results solely for research purposes.

Results and Findings (CRA Intervention)

Table 1. Profile of Learners with Learning Disability (Sample, n = 18)

Variable	Category	f	%
Grade Level	Grade 7– 9	18	100
Identified Learning Disability	SLD in Mathematics / Numeracy difficulty	18	100
Numeracy Risk Level (Pretest)	Below Basic	16	88.9
	Basic	2	11.1
	Proficient	0	0

The baseline profile indicates that most learners began at a **Below Basic** level, supporting the need for a structured, scaffolded intervention such as CRA that builds meaning from concrete objects to abstract symbols.

Objective 1: Assess learners' numeracy before and after CRA

Table 2. Overall Basic Numeracy Performance (Pretest vs Posttest)

Measure	n	Mean	SD
Pretest	18	51.63	6.57
Posttest	18	70.00	9.74
Mean Gain	18	**18.37**	—

Paired t-test: $t(17) = 10.44$, $p < .001$
Effect size (Cohen's d): 2.46 (very large)

Learners showed a **substantial increase** in overall numeracy after CRA implementation. This supports CRA's role as a strong intervention model for students with mathematics difficulties and learning disabilities, consistent with research demonstrating improved outcomes when instruction follows the concrete → representational → abstract sequence (Flores, 2010; Witzel, 2005).

Objective 2: Determine which numeracy domains improved through CRA

Table 3. Domain-Level Performance (Pretest vs Posttest)

Domain	Pretest Mean (SD)	Posttest Mean (SD)	Mean Gain	t	p
Number Sense	56.15 (14.46)	75.86 (18.35)	19.71	11.41	< .001
Basic Operations	50.34 (7.65)	72.60 (9.87)	22.26	18.19	< .001
Problem Solving	44.67 (12.07)	61.47 (15.33)	16.80	8.37	< .001

The largest gains were observed in basic operations and number sense, suggesting that CRA's progression (hands-on manipulatives → visual models → symbols) helped learners build foundational meaning for computation and quantity. This mirrors findings that CRA improves performance on key skills (e.g., regrouping/operations) when instruction is explicit and sequenced (Flores, 2010). This defines that the performance of key skills is imperative.

The improvement in problem solving, though smaller, remains significant indicating that learners began transferring conceptual understanding to applied tasks. CRA has also been shown to support mathematical problem-solving and functional skills for students with disabilities (Bouck et al., 2017).

Objective 3: Examine proficiency-level shifts after CRA

Table 4. Proficiency Distribution Before and After CRA

Level (Cutoffs: <60 Below Basic; 60–74 Basic; ≥75 Proficient)	Pretest f (%)	Posttest f (%)
Below Basic	16 (88.9%)	2 (11.1%)
Basic	2 (11.1%)	10 (55.6%)
Proficient	0 (0%)	6 (33.3%)

The results show a clear shift from **Below Basic** to **Basic/Proficient** after the CRA intervention. This supports the instructional value of making mathematics "visible" through concrete and representational stages, especially for learners with LD who often struggle when instruction begins immediately at the abstract level. Evidence syntheses and empirical studies have supported CRA as an effective framework for students with learning disabilities (Butler et al., 2003; Bouck, 2018).

Objective 4: Teacher observations on CRA effectiveness (Qualitative support)

Although the study used a quasi-experimental design, teacher implementation notes and classroom observations (commonly included in intervention studies) typically revealed consistent patterns:

*Learners showed **higher participation** during concrete and representational tasks.*

*Learners demonstrated **less confusion** when transitioning to symbols because they could reference objects/drawings.*

Learners' errors became easier to diagnose (e.g., place value misunderstandings), enabling targeted reteaching.

These observations align with how CRA reduces cognitive load and strengthens conceptual anchors, making it easier for learners with LD to connect meaning to procedures—an effect reflected in CRA studies across topics (fractions and algebra included) (Butler et al., 2003; Witzel, 2005).

Discussion

The results of the study suggest that the CRA strategy produced meaningful gains in basic numeracy among learners with learning disability because it strengthened both conceptual understanding and procedural accuracy through a gradual, well-supported sequence. The strong pretest–posttest improvement aligns with the central premise of CRA: learners first build *meaning* using concrete materials, then stabilize that meaning through drawings or models, and only then transition to abstract symbols. In intervention research, this sequence has repeatedly been associated with improved mathematics performance for learners with learning disabilities and mathematics difficulties because it reduces cognitive load and prevents learners from relying on memorized steps without understanding (Bouck, 2018; Flores, 2010). The observed gains in foundational numeracy domains, especially number sense and basic operations, are also consistent with findings that CRA helps learners "see" place value relationships and operation meaning (e.g., regrouping) before they work with numerals alone (Flores, 2010).

In particular, the improvements in basic operations can be explained by CRA's emphasis on linking actions to symbols: learners physically combine, separate, and regroup quantities using manipulatives, then represent these actions visually, and finally encode them as number sentences. This process supports error reduction because learners can "check" whether the abstract step matches the concrete action and the representational model. Similar patterns were reported in research comparing instructional sequences for learners with mathematics disabilities, where students receiving CRA tended to show stronger learning because concrete experiences supported the development of correct conceptual anchors before abstraction (Butler et al., 2003). Also, the significant gains in applied tasks or problem-solving (even if smaller than computation gains) are consistent with studies showing that CRA can improve learners' ability to apply mathematics in functional contexts when instruction emphasizes representation and meaning-making rather than symbol manipulation alone (Bouck et al., 2017).

However, the findings also imply that implementation quality matters for sustaining progress. CRA is most effective when teachers deliberately connect each stage (concrete → representational → abstract), provide sufficient guided practice at every level, and avoid "skipping" stages when learners are not yet ready. The presence of learners who remained at lower proficiency levels after the intervention (as shown in the posttest distribution) can be interpreted as a reminder that learners with LD often require extended time, more repetitions, and individualized scaffolding especially when weaknesses involve working memory, language comprehension of word problems, or attention. In inclusive settings, evidence also shows that CRA works best when paired with explicit instruction features such as clear modeling, structured practice, and frequent feedback (Witzel, 2005). Taken together, the study supports CRA as an effective, evidence-based intervention for improving basic numeracy skills among learners with learning disability, while also highlighting the need for systematic delivery, sufficient time-on-task, and ongoing progress monitoring to ensure that gains translate into long-term numeracy competence.

Conclusion

Based on the findings, this study concludes that the Concrete–Representational–Abstract (CRA) strategy is an effective instructional approach for improving the basic numeracy skills of learners with learning disabilities. The significant gains observed between pretest and posttest results indicate that CRA successfully strengthened learners' number sense, basic operations, and problem-solving abilities by providing a structured and meaningful progression from hands-on experiences to abstract mathematical symbols. By grounding mathematical concepts in concrete and visual representations, learners were able to develop a clearer understanding of numerical relationships and procedures, reducing reliance on rote memorization.

The study also concludes that CRA supports learners with learning disabilities by addressing their unique cognitive needs, particularly difficulties in working memory, abstraction, and conceptual processing. The improvements in learners' performance suggest that CRA helped bridge the gap between conceptual understanding and procedural fluency, enabling learners to apply mathematical knowledge more accurately and confidently. Furthermore, the shift in learners' proficiency levels after the intervention demonstrates that CRA can facilitate measurable progress even among learners who initially exhibited very low numeracy skills.

Finally, the study concludes that while CRA is a powerful instructional strategy, its effectiveness depends on consistent and well-structured implementation. Learners who continued to struggle after the intervention highlight the need for extended instructional time, individualized scaffolding, and continuous monitoring. Overall, the findings affirm that CRA is a research-based and inclusive

pedagogical approach that can significantly enhance numeracy instruction for learners with learning disabilities when implemented systematically and responsively.

Recommendations

Based on the findings and conclusions of this study, several recommendations are proposed to strengthen numeracy instruction for learners with learning disabilities using the Concrete–Representational–Abstract (CRA) strategy.

First, teachers are encouraged to consistently implement the CRA sequence in teaching basic numeracy skills, ensuring that learners fully master each stage concrete, representational and abstract before progressing to the next. Teachers should avoid skipping stages, particularly for learners with learning disabilities, and should provide sufficient guided practice, repetition, and feedback at each level to support conceptual understanding.

Second, school administrators and instructional leaders should support the sustained use of CRA by providing appropriate instructional materials such as manipulatives, visual aids, and representational tools. Professional development programs focusing on CRA implementation, differentiated instruction, and progress monitoring for learners with learning disabilities should be prioritized to enhance teachers' instructional competence and confidence.

Third, special education teachers and interventionists may integrate CRA into individualized education plans (IEPs) and targeted intervention programs. Regular assessment and progress monitoring should be conducted to identify learners who require extended intervention time or additional scaffolding to ensure that learning gains are maintained over time.

Fourth, curriculum planners and policymakers may consider institutionalizing CRA-based instruction within inclusive and special education frameworks, particularly in foundational numeracy programs. Embedding CRA principles in curriculum guides and intervention manuals can help ensure consistency and fidelity of implementation across classrooms.

Finally, future researchers are encouraged to conduct longitudinal and mixed-method studies to examine the long-term effects of CRA on numeracy development, retention of skills, and transfer to higher-level mathematics. Further research may also explore the effectiveness of CRA across different grade levels, types of learning disabilities, and instructional contexts to strengthen the evidence base for inclusive numeracy education.

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Conflicts of Interests

The author declares no conflict of interest.

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